B.Sc - part-II (Sub) Diff. Calculus. 07.07.2020

Problems based on the relation between Derivatives of Different Order.

Q.1. If $y = a\sin(\log x)$, prove that

$$x^2y_2 + xy_1 + y = 0$$

Soln: $\because y = a\sin(\log x)$

Diff. with respect to x, we get

$$y_1 = a\cos(\log x)\frac{1}{x}$$

$$\Rightarrow xy_1 = a\cos(\log x)$$

Again, diff. w.r. to x, we obtain

$$xy_2 + y_1 = -a\sin(\log x)\frac{1}{x} = -\frac{y}{x}$$

$$\Rightarrow x^2y_2 + xy_1 = -y$$

$$\Rightarrow x^2y_2 + xy_1 + y = 0 \text{ proved.}$$

Q.2. If $x = \cos(\log y)$, show that

$$(1-x^2)y_2 - xy_1 = y.$$

Soln: We have, $x = \cos(\log y)$.

$$\therefore \log y = \cos^{-1}x$$

Differentiating w.r. to x, we get

$$\frac{1}{y}y_1 = -\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{y_1}{y} = -\frac{1}{\sqrt{1-x^2}}$$

Squaring both sides, we get

$$\frac{y_1^2}{y^2} = \frac{1}{1-x^2}$$

$$\Rightarrow (1-x^2)y_1^2 = y^2$$

[illegible] ~~Part~~ – II (Sub). ~~Set Theory~~. 07-07-2020

Diff. Calculus.

Rest part of Q. 2

Again, diff. w.r. to x, we get

$$(-2x)y_1^2 + (1-x^2)2y_1y_2 = 2yy_1$$

$$\Rightarrow (1-x^2)y_2 - xy_1 = y$$

Proved.

Q. 3: If $y = a\cos\log x$, prove that

$$x^2y_2 + xy_1 + y = 0.$$

Soln: $\because y = a\cos\log x$

Diff. w.r. to x, we get

$$y_1 = -a\sin\log x \cdot \frac{1}{x}$$

$$\Rightarrow xy_1 = -a\sin\log x$$

$$\Rightarrow \frac{xy_1}{a} = -\sin\log x$$

$$= -\sqrt{1-\cos^2\log x} = -\sqrt{1-\frac{y^2}{a^2}}$$

Squaring both sides, we get

$$\frac{x^2y_1^2}{a^2} = 1 - \frac{y^2}{a^2} = \frac{a^2-y^2}{a^2}$$

$$\Rightarrow x^2y_1^2 = a^2 - y^2$$

Again, diff. w.r. to x, we get

$$2xy_1^2 + x^2 \cdot 2y_1y_2 = -2yy_1$$

$$\therefore x^2y_2 + xy_1 + y = 0$$

Proved.

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